

# "Skill" Ranking in Memoir '44 Online

## Introduction

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This document describes the "Skill" ranking system used in Memoir '44 Online *as of beta 13*. Even though some parts are more suited to the mathematically inclined reader, we try to keep the basic concepts as clear as possible and emphasize that whatever the solution, it is no easy matter!

## General Constraints

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Any ranking/ladder system is always the subject of hot discussions in games and sports: the Chess ELO system, the Bridge competition, the tennis ATP system, etc. To put it simply, there is no perfect solution, no matter what. The notion of "fair" is highly subjective to each player, depending on his/her vision of the game - which also evolves with time and experience.

- **Non-linear spread:** most players are average, and only a few are exceptionally good (or bad) players. So the skill system should reflect this. The original Chess ELO is good at this.
- **Converge quickly, evolve slowly:** basically, these are two opposite goals! The basic idea is that a player enters the competition with a given "level" of expertise. So you need to find quickly where he should fit in the ladder. Then over time, his expertise will evolve (usually grow), and therefore he should move (usually up) slowly.
- **Inertia to account for the luck factor:** if, after long time and efforts, a player reaches a high level, he will probably get mad to fall down abruptly because of bad rolls in a game. Such punishment should be limited, but at the same time, good players should not be protected in some kind of ivory tower. Chess ELO is bad at this, which is why the Chess people invented groups (Master, Grand Master, etc.).
- **Don't reward best players for killing newcomers:** it should be considered normal for a good player to beat a bad player, so the reward should be minimized. Chess ELO is good at this.
- **Allow entry at any time:** this is probably the constraint that causes the most headaches. If all players start at the same time and all play, then it's easy. It's like any tournament or sport competition (take the European Football league for example). Here, we have players that join the fray. Chess ELO is OK at this if you wait for a while for the score to "converge".
- **Newcomers-compatible:** if the number of cumulated Skill points keeps growing with the number of games played, newcomers will never be able to catch-up. Chess ELO is good at addressing this issue.

On top of this, Memoir '44 brings some unique constraints:

- **Scenarios are unbalanced:** the system should take into account the risk vs. reward factor.
- **Need for symmetry:** exact same scores during a match and rematch should put the two players where they were before they played the two games.
- **Reward the winner:** the winner should always make points, even if he is on the favored side. We feel that doing it another way would go against the Memoir '44 spirit.
- **Reward over-performance:** if the losing player performs better than the average, he should be rewarded too. This is also part of the Memoir '44 spirit: the win margin should make a difference. This is also important to avoid the "rage quit" behavior that plagues most online games nowadays, i.e. players who quit games before the end when they start to lose.
- **There is always a winner:** it sounds funny put like this, but this is quite different from Chess. With Chess ELO, players of same expertise are expected to end up with a draw, and therefore no points won or lost. In Memoir '44, there will always be a winner.

## The Current ELO System in Memoir '44 Online

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We had to start with something. So we took the ELO system that was used in Ticket to Ride Online (even though we knew it was far from perfect), and worked on addressing the balanced and over-performance constrains.

Roughly, our current ELO system is a classic one. During a game between two players, a given number of points is at stake. The winner's score is increased by this amount, and the loser's score is decreased by the same amount. So it is a zero-sum system.

The amount of points at stake depends on the spread between the two players and on who won/lost. If the winner is the best ranked, his win is considered normal, so less points are at stake. If the best ranked player lose, much more points are at stake ("correction"). This number of points is even higher if the players were very far apart.

The number of points also follows a "bell curve".

People enter the system by the middle. We consider that their score has converged after 20 games, which is the required number of games to enter the Leaderboard. They also drop out of the Leaderboard after 15 days of inactivity.

## Main Formulae

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We keep the basic principles of the classic ELO system:

- People enter the table with the average 1,500 score.
- Zero-energy rule: the same number of points is added (or subtracted) to the winner (the loser).
- A bell-curve damping function is applied.

### The original ELO formulae:

$$r'_A = r_A + pts \times \left( s(sc_A, sc_B) - \frac{1}{1 + 10^{\frac{r_B - r_A}{400}}} \right)$$

$$\text{with } s() \text{ function as: } s(sc_A, sc_B) = \begin{cases} 1 & \text{if } sc_A > sc_B \quad (A \text{ wins}) \\ 0.5 & \text{if } sc_A = sc_B \quad (\text{tie}) \\ 0 & \text{if } sc_A < sc_B \quad (B \text{ wins}) \end{cases}$$

$r_A$  : rank value of player A       $r'_A$  : new rank value

$sc_A$  : score of player A       $sc_B$  : score of player B

$pts = 32$

### New concepts:

We only change the number of points at stake. The pts constant of 32 points becomes a function that uses the *Win/Loss Ratio*. The idea is that if you play the Allies on a scenario where they win two third of the time, then you should get only one third of the original points if you win. But if the Axis win, then they get two third of the points. In other words, the number of points at stake exactly balances the win/loss probability. You have fewer chances to win with the Axis, but you make more points if you do.

$$pts(sc_A, sc_B, ratio) = 64[s(sc_A, sc_B) \times (1 - ratio) + (1 - s(sc_A, sc_B)) \times ratio]$$

After simplification:

$$pts(sc_A, sc_B, ratio) = 64[s(sc_A, sc_B) + ratio - 2 \times ratio \times s(sc_A, sc_B)]$$

There is one last finishing touch left: we want to reward the winner if he won with a high margin or not. The idea is simple; it depends on how far the loser is from his side's Average Score. If he is 1 medal behind, the winner gets a 10% bonus, 2 medals give 20%, etc. On the other end, if the loser did better than his side's average, he reduces the winners' points by 10% for one better medal, 20% for 2, etc.

This gives:

$$bonus(sc_A, sc_B, av_A, av_B) = 1 + 0.1 \times s(sc_A, sc_B) \times (av_B - sc_B) + 0.1 \times (1 - s(sc_A, sc_B)) \times (av_A - sc_A)$$

with  $av_A$  the average score in player A's side

and  $av_B$  the average score in player B's side

## Complete formulae:

$$r'_A = r_A + pts(sc_A, sc_B, ratio) \times bonus(sc_A, sc_B, av_A, av_B) \times \left( s(sc_A, sc_B) - \frac{1}{1 + 10^{\frac{r_B - r_A}{400}}} \right)$$

with:

$$s(sc_A, sc_B) = \begin{cases} 1 & \text{if } sc_A > sc_B \quad (A \text{ wins}) \\ 0.5 & \text{if } sc_A = sc_B \quad (tie) \\ 0 & \text{if } sc_A < sc_B \quad (B \text{ wins}) \end{cases}$$

$$pts(sc_A, sc_B, ratio) = 64[s(sc_A, sc_B) + ratio - 2 \times ratio \times s(sc_A, sc_B)]$$

$$bonus(sc_A, sc_B, av_A, av_B) = 1 + 0.1 \times s(sc_A, sc_B) \times (av_B - sc_B) + 0.1 \times (1 - s(sc_A, sc_B)) \times (av_A - sc_A)$$

$r_A$  : rank value of player A       $r'_A$  : new rank value

$sc_A$  : score of player A       $sc_B$  : score of player B

$av_A$  : average score on A's side     $av_B$  : average score on B's side

$ratio$  : Win/Loss Ratio of A's side vs. B's side

More examples are computed in Appendix I. They show various situations on 3 typical scenarios. We show only computation for players of same ranking: since the ELO part did not change, there was no point in creating another variable axis.

## APPENDIX I

### Scenario Scores Examples

**Pegasus Bridge:** *Usually a British victory, but by a short margin*

Battles fought: 1571

Victory Conditions: 4 medals

| Sides   | Victories | Ratio | Average Score | Standard Deviation | Distance bet. Averages |
|---------|-----------|-------|---------------|--------------------|------------------------|
| Allies  | 1071      | 68%   | 3.4           | 2.8                | 0.8                    |
| Germans | 500       | 32%   | 2.6           | 1.4                |                        |

**Omaha Beach:** *Germans crush Allies most of the time*

Battles fought: 686

Victory Conditions: 6 medals

| Sides   | Victories | Ratio | Average Score | Standard Deviation | Distance bet. Averages |
|---------|-----------|-------|---------------|--------------------|------------------------|
| Allies  | 142       | 21%   | 3.1           | 1.7                | 2.4                    |
| Germans | 544       | 79%   | 5.5           | 2.0                |                        |

**Operation Cobra:** *A well-balanced scenario*

Battles fought: 484

Victory Conditions: 5 medals

| Sides   | Victories | Ratio | Average Score | Standard Deviation | Distance bet. Averages |
|---------|-----------|-------|---------------|--------------------|------------------------|
| Allies  | 233       | 48%   | 3.8           | 1.7                | 0.0                    |
| Germans | 251       | 52%   | 3.8           | 1.7                |                        |

## APPENDIX II

### Ranking Data Simulations

|  | rA   | rB   | Medals Needed | W/L ratio | av1 | av2 | score A | score B | s    | pts   | bonus | ptsA   | ptsB   | r'A     | r'B     |
|--|------|------|---------------|-----------|-----|-----|---------|---------|------|-------|-------|--------|--------|---------|---------|
| ELO standard                             | 1500 | 1500 |               |           |     |     | 4       | 2       | 1.00 | 32.00 | 1.00  | 16.00  | -16.00 | 1516.00 | 1484.00 |
| <b>Pegasus Bridge (A playing Allies)</b> |      |      |               |           |     |     |         |         |      |       |       |        |        |         |         |
| Equal players, A wins, B as expected     | 1500 | 1500 | 4             | 66%       | 3.4 | 2.6 | 4       | 2       | 1.00 | 21.76 | 1.06  | 11.53  | -11.53 | 1511.53 | 1488.47 |
| Equal players, A wins, B better          | 1500 | 1500 | 4             | 66%       | 3.4 | 2.6 | 4       | 3       | 1.00 | 21.76 | 0.96  | 10.44  | -10.44 | 1510.44 | 1489.56 |
| Equal players, A wins, B lower           | 1500 | 1500 | 4             | 66%       | 3.4 | 2.6 | 4       | 1       | 1.00 | 21.76 | 1.16  | 12.62  | -12.62 | 1512.62 | 1487.38 |
| Equal players, A wins, B crushed         | 1500 | 1500 | 4             | 66%       | 3.4 | 2.6 | 4       | 0       | 1.00 | 21.76 | 1.26  | 13.71  | -13.71 | 1513.71 | 1486.29 |
| Equal players, A loses                   | 1500 | 1500 | 4             | 66%       | 3.4 | 2.6 | 2       | 4       | 0.00 | 42.24 | 1.14  | -24.08 | 24.08  | 1475.92 | 1524.08 |
| Equal players, A loses better            | 1500 | 1500 | 4             | 66%       | 3.4 | 2.6 | 3       | 4       | 0.00 | 42.24 | 1.04  | -21.96 | 21.96  | 1478.04 | 1521.96 |
| Equal players, A loses -                 | 1500 | 1500 | 4             | 66%       | 3.4 | 2.6 | 1       | 4       | 0.00 | 42.24 | 1.24  | -26.19 | 26.19  | 1473.81 | 1526.19 |
| Equal players, A crushed                 | 1500 | 1500 | 4             | 66%       | 3.4 | 2.6 | 0       | 4       | 0.00 | 42.24 | 1.34  | -28.30 | 28.30  | 1471.70 | 1528.30 |
| <b>Omaha Beach (A playing Germans)</b>   |      |      |               |           |     |     |         |         |      |       |       |        |        |         |         |
| Equal players, A wins, B better+         | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 6       | 5       | 1.00 | 13.44 | 0.81  | 5.44   | -5.44  | 1505.44 | 1494.56 |
| Equal players, A wins, B better          | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 6       | 4       | 1.00 | 13.44 | 0.91  | 6.12   | -6.12  | 1506.12 | 1493.88 |
| Equal players, A wins, B as expected     | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 6       | 3       | 1.00 | 13.44 | 1.01  | 6.79   | -6.79  | 1506.79 | 1493.21 |
| Equal players, A wins, B lower+          | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 6       | 2       | 1.00 | 13.44 | 1.11  | 7.46   | -7.46  | 1507.46 | 1492.54 |
| Equal players, A wins, B lower++         | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 6       | 1       | 1.00 | 13.44 | 1.21  | 8.13   | -8.13  | 1508.13 | 1491.87 |
| Equal players, A wins, B crushed         | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 6       | 0       | 1.00 | 13.44 | 1.31  | 8.80   | -8.80  | 1508.80 | 1491.20 |
| Equal players, A loses                   | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 5       | 6       | 0.00 | 50.56 | 1.05  | -26.54 | 26.54  | 1473.46 | 1526.54 |
| Equal players, A loses better            | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 4       | 6       | 0.00 | 50.56 | 1.15  | -29.07 | 29.07  | 1470.93 | 1529.07 |
| Equal players, A loses                   | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 3       | 6       | 0.00 | 50.56 | 1.25  | -31.60 | 31.60  | 1468.40 | 1531.60 |
| Equal players, A loses better            | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 2       | 6       | 0.00 | 50.56 | 1.35  | -34.13 | 34.13  | 1465.87 | 1534.13 |
| Equal players, A loses -                 | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 1       | 6       | 0.00 | 50.56 | 1.45  | -36.66 | 36.66  | 1463.34 | 1536.66 |
| Equal players, A crushed                 | 1500 | 1500 | 6             | 79%       | 5.5 | 3.1 | 0       | 6       | 0.00 | 50.56 | 1.55  | -39.18 | 39.18  | 1460.82 | 1539.18 |

| <b>Operation Cobra (A playing Allies)</b> | <b>rA</b> | <b>rB</b> | <b>Medals Needed</b> | <b>W/L ratio</b> | <b>av1</b> | <b>av2</b> | <b>score</b> |          | <b>s</b> | <b>pts</b> | <b>bonus</b> | <b>ptsA</b> | <b>ptsB</b> | <b>r'A</b> | <b>r'B</b> |
|---|-----------|-----------|----------------------|------------------|------------|------------|--------------|----------|----------|------------|--------------|-------------|-------------|------------|------------|
|   |           |           |                      |                  |            |            | <b>A</b>     | <b>B</b> |          |            |              |             |             |            |            |
| Equal players, A wins, B better           | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 5            | 4        | 1.00     | 33.28      | 0.98         | 16.31       | -16.31      | 1516.31    | 1483.69    |
| Equal players, A wins, B as expected      | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 5            | 3        | 1.00     | 33.28      | 1.08         | 17.97       | -17.97      | 1517.97    | 1482.03    |
| Equal players, A wins, B lower+           | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 5            | 2        | 1.00     | 33.28      | 1.18         | 19.64       | -19.64      | 1519.64    | 1480.36    |
| Equal players, A wins, B lower++          | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 5            | 1        | 1.00     | 33.28      | 1.28         | 21.30       | -21.30      | 1521.30    | 1478.70    |
| Equal players, A wins, B crushed          | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 5            | 0        | 1.00     | 33.28      | 1.38         | 22.96       | -22.96      | 1522.96    | 1477.04    |
| Equal players, A loses better             | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 4            | 5        | 0.00     | 30.72      | 0.98         | -15.05      | 15.05       | 1484.95    | 1515.05    |
| Equal players, A loses                    | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 3            | 5        | 0.00     | 30.72      | 1.08         | -16.59      | 16.59       | 1483.41    | 1516.59    |
| Equal players, A loses -                  | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 2            | 5        | 0.00     | 30.72      | 1.18         | -18.12      | 18.12       | 1481.88    | 1518.12    |
| Equal players, A loses --                 | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 1            | 5        | 0.00     | 30.72      | 1.28         | -19.66      | 19.66       | 1480.34    | 1519.66    |
| Equal players, A crushed                  | 1500      | 1500      | 5                    | 48%              | 3.8        | 3.8        | 0            | 5        | 0.00     | 30.72      | 1.38         | -21.20      | 21.20       | 1478.80    | 1521.20    |